A theory for the radiation pattern of a narrow-strip acoustic transducer

A. R. Selfridge, G. S. Kino, and B. T. Khuri-Yakub
Ginzton Laboratory, Stanford University, Stanford, California 94305

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An important criterion in the design of transducer array elements for acoustic imaging is the angular response, or the far-field radiation pattern, of a single element. In this letter, we show that the widely accepted formula for the angular response function is inadequate and must be multiplied by $\cos\theta$. Good agreement with experiment is then obtained.

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Acoustic transducer arrays are being widely used in electronically scanned and focused imaging systems. One important criterion for their operation is that of the angular response of an individual array element. It has been common practice to assume that the angular response or far-field radiation pattern of a single element is of the form

$$p = p_0 \frac{\sin(\pi w/\lambda \sin\theta)}{(\pi w/\lambda \sin\theta)}, \tag{1}$$

where p is the pressure due to the acoustic field, w is the width of the transducer element, and λ is the wavelength of the acoustic wave in the propagating medium. We suggest that this formula has been arrived at by an inadequate interpolation of scalar diffraction theory because it is usually assumed that the transducer element is surrounded by a rigid baffle.

We consider the array element shown in Fig. 1, emitting an acoustic wave into a liquid. We assume the pressure due to the array element $p_0(x)$ is uniform over a width w and is zero outside the transducer. In typical applications, the transducer element is separated from the water medium by a thin membrance or by a rubberlike material of the same impedance as water. Thus the assumption that $p_0(x) = 0$ for $|x| > \frac{1}{2}w$ would seem to be a far better approximation than the alternative assumption commonly used in acoustic theory that the transudcer is surrounded by a rigid baffle.[1]

Since the pressure obeys the scalar wave equation, we could use the normal three-dimensional Rayleigh-Sommerfeld diffraction formula[2] and write

$$p(r,\theta) = \frac{1}{j\lambda} \int p_0(x,y) \frac{e^{2j\pi R/\lambda}}{\mathbf{R}} \cos\theta' \, dx \, dy, \tag{2}$$

where R is the distance between the source point $(x,0)$ and the observation point (r,θ), θ' is the angle between the radius vector $\mathbf{R}$ and the normal to the transducer, and $p_0(x)$ is the pressure at the plane of the transducer. As we are interested in a two-dimensional system taken to be infinite in the x direction, the Green's function required has the form of a Hankel function. For $2\pi R/\lambda \gg 1$, the required form of Eq. (2) becomes

$$p(r,\theta) = \frac{1}{j\lambda^{1/2}} \int_{-\frac{1}{2}w}^{\frac{1}{2}w} p_0(x) \frac{e^{2j\pi R/\lambda}}{\mathbf{R}^{1/2}} \cos\theta' \, dx.$$

For points in the far field, we can assume that $\theta' \approx \theta$ and $R \approx r$, as far as its effect on the amplitude concerned. Where the phase variation is concerned, we write $R \approx r - x\sin\theta$. This leads to the result

$$p(r,\theta) = \frac{p_0 w}{j(\lambda)^{1/2}} e^{2j\pi r/\lambda} \frac{\sin(\pi w/\lambda \sin\theta)}{\pi w/\lambda \sin\theta} \cos\theta. \tag{3}$$

We note that because of the use of the Rayleigh-Sommerfeld formula, there is an extra $\cos\theta$ term. The result implies therefore that however narrow the width of the transducer, the response must fall to zero at $\theta = \frac{1}{2}\pi$ and fall off monotonically with angle θ, a result which would certainly seem to be physically reasonable.

We have checked this result against a number of experiments of our own and others. Figure 2 illustrates an experiment carried out by DeSilets on a 2.5-MHz transducer with a transducer element of width 0.373 mm. The full line shows the corrected theory and the dashed line the uncorrected theory, with the $\cos\theta$ term missing. We have seen that the agreement between experiment and theory is excellent, using

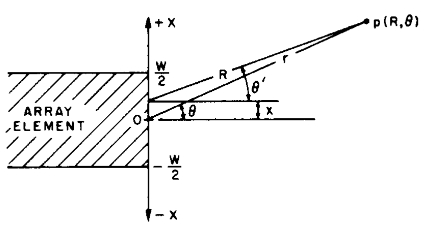

FIG. 1. Schematic diagram of any array element radiating into water.

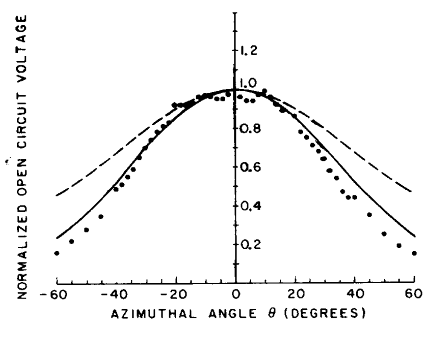

FIG. 2. Angular response data taken by DeSilets (Ref. 4) compared to present theory (solid line) and old theory (broken line). Element was 0.373 mm wide operating at 3 MHz into water.

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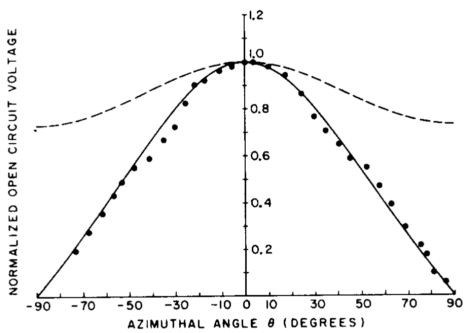

FIG. 3. Angular response data taken by present authors compared to present (solid line) and old theory (broken line). Element width was 0.305 mm operating at 2.5 MHz into water.

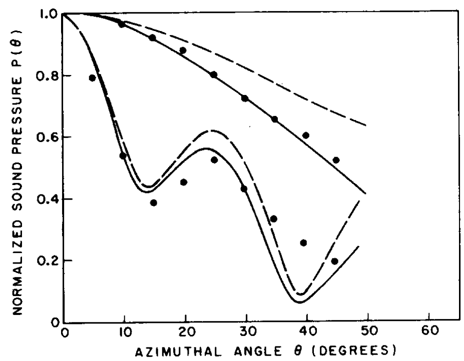

FIG. 4. Recently published work by Sato *et al.* (Ref. 3) showing measurements (data) compared to their theory (broken line). When the predicitons of the Sato theory are multiplied by cos θ, the solid line is obtained. The upper curves correspond to an element 0.361 mm wide operating at 2.5 MHz into water ($h/\lambda = 0.6$). The lower curve is for an element 1.6 mm wide operating at 2.5 MHz into water ($h/\lambda = 2.67$).

the new theory. A similar set of results taken with a narrower transducer element by the present authors gives correspondingly good agreement, as shown in Fig. 3. Finally, in Fig. 4, we refer to some recently published work by Sato[3] where we have corrected his theoretical curves by multiplying them by $\cos\theta$. Again, the agreement between theory and experiment is excellent for the narrower element, for which Eq. (3) would do as well. For a very wide element, however, the agreement is not as good at large angles. We believe that this effect is due to the fact that the pressure at the transducer surface is nonuniform, as would be expected from Sato's numerical results, and Sato has used the displacement u_{z0} rather than the pressure $p_0(y)$ in his theory.

Perhaps a still more rigorous approach for a high-impedance transducer material facing a low-impedance liquid would be to assume that the displacement u_{z0} is uniform over the transducer face, with $p_0(x) = 0$ for $|x| > \frac{1}{2}w$. However, this boundary condition is more difficult to treat mathematically. For narrow transducer elements, the assumption of a uniform pressure field at the face of the transducer is a reasonable one, while for wide elements it is not clear that either assumption would be ideal.

In conclusion, by use of the proper interpretation of the Rayleigh-Sommerfeld theory, we have corrected some anomalies in the experimental results obtained form narrow-strip transducers.

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