

# A QUANTITATIVE SCHLIEREN METHOD FOR THE INVESTIGATION OF AXISYMMETRICAL SHOCK WAVES

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## Abstract

Schlieren methods are well established to visualize acoustical, thermodynamic, and other phenomena, which influence the refractive index distribution in transparent media. We propose a quantitative schlieren method for the investigation of an ultrasonic field distribution generated by an electromagnetic (EM) shock wave source. In our schlieren system we have used a shiftable slit with variable width for sampling the angular spectrum of diffracted light. To compute the pressure distributions of shock waves a tomographical concept has been utilized. We discuss accuracy and limitations of the schlieren method and compare reconstruction results with sensor measurements.

## Introduction

Schlieren methods have reached their highest development in the study of processes in gases because of the high compressibility and the accompanying large refractive index variations of these media [4]. In hydrodynamic investigations a relatively high pressure gradient is necessary to apply these methods. In the axisymmetrical shock wave fields generated by an EM source the pressure gradients are sufficiently high to utilize the usual schlieren methods. Typically a light beam penetrating the lithotripter wave in the focal area perpendicularly with respect to the direction of propagation will yield a deflection angle in the range of  $\pm 1^\circ$ . Nevertheless, up to these days only qualitative schlieren methods have been realized for the investigation of the lithotripter waves [6].

We have sampled the light deflection distribution due to the shock wave field by means of a narrow slit in the focal plane of the analyzing lens (lens L3, see Fig. 1 in [1]). The deflection angle resolution can be improved by reducing the width of the slit. However, light diffraction at the slit leads to a lower limit for the slit size. As the diffraction fringes not only depend on the slit width but also on the local field distribution

it is necessary to adapt the slit size for each deflecting point.

From the light deflection distribution we compute the refractive index gradient. Several numerical methods are known for the computation of the refractive index gradient of axisymmetrical objects from deviation angles [3], [5], [8]. Mostly they use a piecewise polynomial interpolation of the gradient distribution or of a function of the gradient distribution in each cross section perpendicular to the axis of symmetry. However, for an appropriate accuracy these methods are computationally not efficient. In this paper we propose a different approach utilizing a tomographical concept. With our method the computation of the pressure field distribution from the measured data is feasible with lower computational effort.

## Measurement principle

Quantitative schlieren methods evaluate the refractive index distribution inside a test section from the light deflection angle distribution. Detection of the light deviation angles in only one direction, e.g. in the nominal shock wave direction, is sufficient for the computation of the refractive index as well as the pressure field distribution. We have detected the vertical deviations of light of upward propagating shock waves, because they are considerably larger than the horizontal deviations. For this purpose we have used the same experimental setup as described in [1]. The test section was illuminated by collimated light of argon laser (515nm line, max. 2.1 W light power c.w.), which was pulsed by means of two Pockels cells (pulse width 100ns).

The sensibility of a schlieren system is controlled by the size of the light source image and the focal length of the analyzing lens. In our setup the image of the light source was enlarged from  $30\mu\text{m}$  to approximately  $200\mu\text{m}$  mainly due to the poor optical quality of the acryl glass of the experimental basin. Interferences at the protection layer of the CCD-chip limit the focal length of the analyzing lens.

For a parallel beam illumination of the test section the light source image can be seen as the angular spectral decomposition of the transmission function caused by the shock wave. A horizontal slit placed in the image plane of the light source acts as a selective filter with its center frequency given by the slit displacement from the optical axis and the bandwidth given by the slit width. The uncertainty principle limits the detection accuracy of locations and deviation angles. It is obvious that for the purpose of determining of the deviation angle distribution the locations and the deviation angles should be known with comparable accuracy. So we have to optimize the slit width.

Therefore we have imaged the locations of equal light deflection angles, which are the contour lines of the deviation angle distribution function, several times using different slit widths. If the slit size was too small diffraction at the slit broadens the contour lines. Using a wide slit also leads to broad contour lines due to coarse resolution of deflection angles. Hence we compose the resulting contour line of one deflection angle from thin parts of the contour lines obtained by different slit sizes.

In the case of zero deflection determination it is more convenient to use a thin filament instead of a slit in order to obtain better contrast.

Approximately one hundred shock waves have been analyzed by different positions and different widths of the slit to determine the deflection angle distribution of the shock wave in the focus of the lithotripter.

## Evaluation of the pressure field distribution

Consider the following geometry of the experiment, where  $z$  is the nominal shock wave propagation direction and  $x$  the nominal light propagation direction.

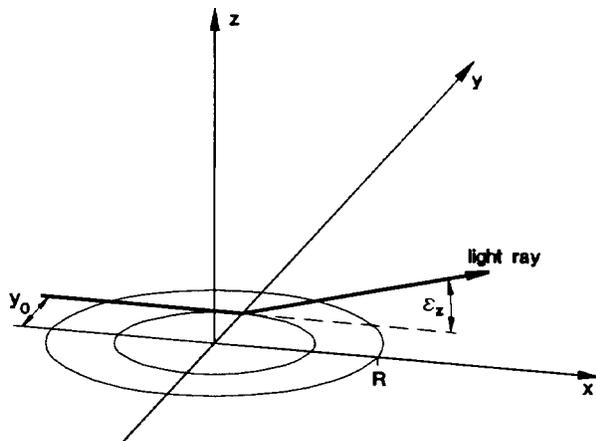


Figure 1. The geometry of the experiment.

According to the geometrical optics approach for small deflection angles, the light deviation angle is given by

$$\epsilon_z = \frac{2}{n_{w0}} \int_{y_0}^R \left( \frac{\partial n_w}{\partial z} \right) \frac{r dr}{\sqrt{r^2 - y_0^2}} \quad (1)$$

where  $n_{w0} = 1.3362$  is the refractive index of water under normal pressure for a light wavelength  $\lambda = 515nm$ .

The line integral can be understood as the parallel beam projection of the z-component of the refractive index gradient. We have utilized a tomographic concept to recover this component of the refractive index gradient from the deflection angle distribution.

The tomographical procedure can be simplified because each projection taken around the axis of symmetry gives the same result. We have applied a filtered backprojection algorithm [2] for computation of the z-component of the refractive index gradient. The refractive index distribution in the test section has been obtained by numerical integration.

Theoretical considerations [7] and experimental investigations [9] show linear dependence between acoustic pressure and refractive index:

$$\frac{p}{[MPa]} = \frac{n_w - n_{w0}}{1.4 \cdot 10^{-4}} \quad (2)$$

On the basis of this dependence the pressure field distribution can be computed from the refractive index distribution.

## Results

Using our practical realization we have evaluated the shock wave pressure field in the focus of an EM lithotripter. The discharge voltage has been set to  $13kV$  and the light pulse has been generated  $261.63 \mu s$  after excitation of the shock wave. Figure 2 demonstrates three steps of the quantitative schlieren method. Figure 2a shows the deflection angle distribution, which has been interpolated from the measured contour lines. The image of the z-component of the refractive index gradient (Fig. 2b) has been computed by means of the tomographical backprojection algorithm. The third step is the numerical integration of the refractive index gradient, using the constant refractive index ( $n_{w0}$ ) in front of the shock wave and the linear dependence between the refractive index and the acoustic pressure.

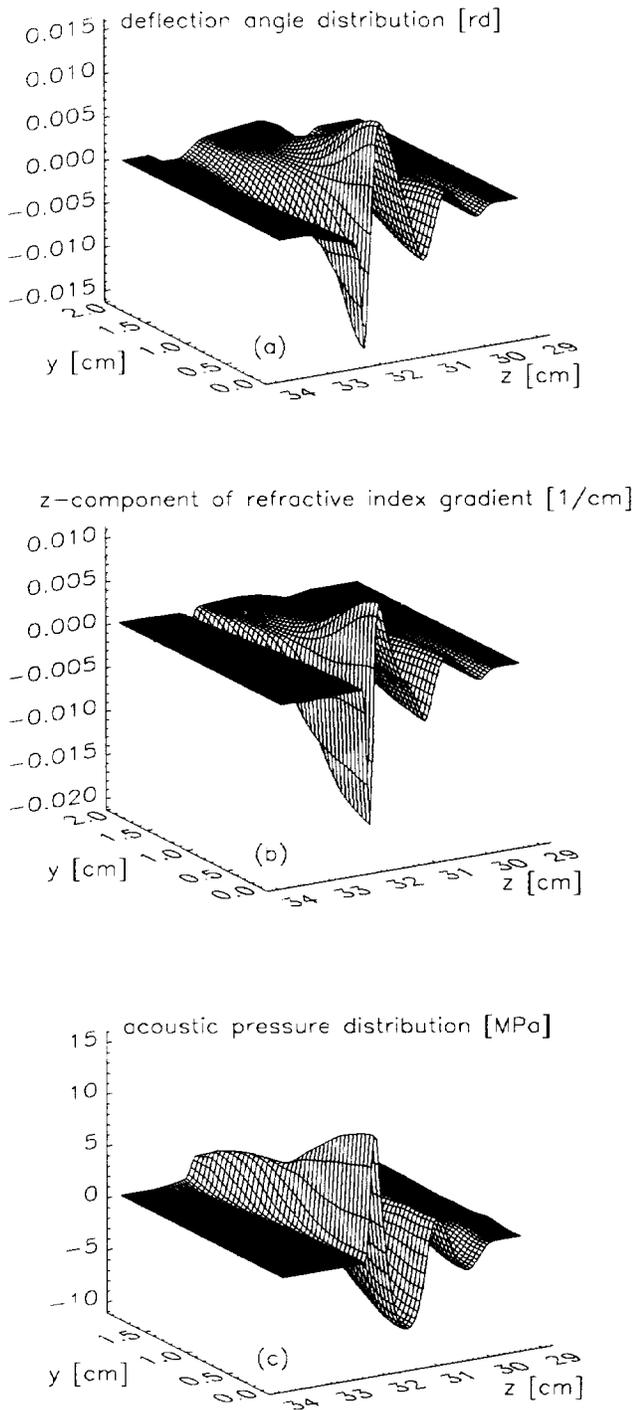


Figure 2. Measured deflection distribution (a), reconstructed refractive index gradient (b) and resulting acoustic pressure distribution (c).

The results were compared with measurements of shock waves with a PVDF membrane hydrophone, which was placed in the centre of the test section on the axis of symmetry.

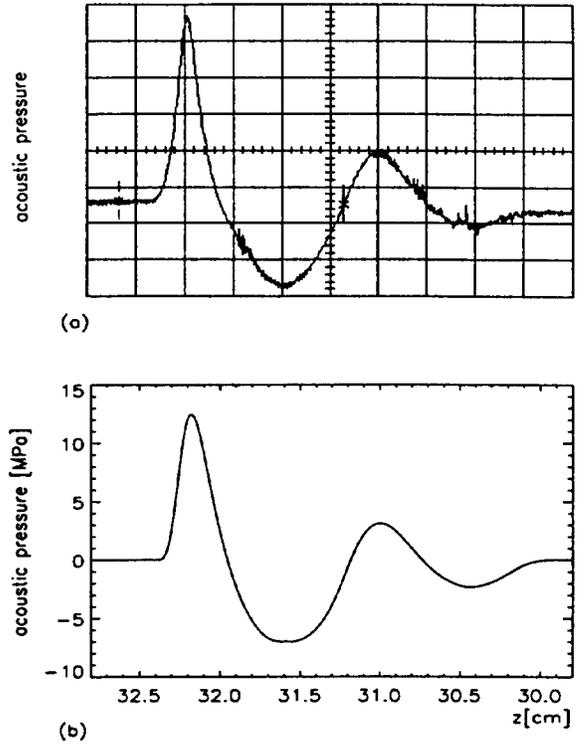


Figure 3. Shock wave pressure on the axis of symmetry measured with the PVDF-membrane hydrophone (a) and reconstructed result from the quantitative schlieren method (b).

From the reconstructed pressure field on the axis the signal at the location of the hydrophone can be estimated if the non-linear steepening within the test-section is neglected. The reconstruction and the hydrophone measurement give similar results (Fig 3). Particular features of the shock wave such as steepening of the shock-front (to the rise time of about 700 ns), the negative pressure amplitude ( $-7\text{MPa}$ ) and the center frequency ( $140\text{kHz}$ ) could be reconstructed.

The feasibility to resolve thinner shock-fronts of higher amplitude waves has been limited by the sensitivity of our system. The sensitivity can be improved by using optical components of better quality for the system (acryl glass basin, CCD camera).

The quantitative schlieren method yields the absolute values of the acoustic pressure field distribution.

So we conclude that this method is a powerful tool to study shock waves in water. The most important application of this method in the future may be testing and calibration of shock wave sensors.

## Acknowledgement

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