FUNDAMENTAL CONCEPTS IN ACOUSTIC TRANSDUCER ARRAY DESIGN

A.R. Selfridge, G.S. Kino, and B.T. Khuri-Yakub

Edward L. Ginzton Laboratory
Stanford University
Stanford, California 94305

Abstract

Ceramic strip resonators with rectangular cross-section have been made with aspect ratios (width to height ratios) from 0.1 to 30. The short and open circuit resonant frequencies have been plotted as a function of aspect ratio and show good agreement between experiment and a simple coupled mode theory.

The effect of loading these resonators acoustically with solids and liquids has also been examined. The resonators we have measured had a center frequency of near 3 MHz and widths down to 0.12 wavelengths in water at this frequency. We have demonstrated for the first time that when the resonators are less than a wavelength in width, a complex acoustic load impedance must be used to accurately predict the electrical impedance of the resonator which is measured.

1. Introduction

The transducer elements required for an acoustic imaging array are normally rectangular in cross-section. Typically, their height is chosen to be larger than their width L so that the transverse resonance is at a considerablly higher frequency than the extensional resonance associated with the height H. This choice of geometry has two effects: (1) the resonant frequency of the principal mode and the effective coupling coefficient depend on both H and L; (2) the width of the transducer may be less than a half wavelength in the medium in which a wave is being excited, as well as in the backing. Consequently, the transducer does not radiate power as efficiently as a very wide transducer, and the effective impedance of the load is complex and often quite different from the longitudinal plane wave impedance of the medium.

In this paper, we report on a set of experiments made to check the Onoe-Tiersten[1] theory for rectangular short circuited and a modified form of their theory for open circuited rectangular resonators over a range of configuration ratios $G = L/H$ varying from 0.1 to 30. In addition, experimental checks are carried out of the variational theory of Kino and DeSilets[2] on the radiation impedance of a finite width resonator into either a solid or a liquid. The theory is further checked for resonators with quarter wavelength matching layers, and good agreement is obtained between theory and experiment.

2. The Unloaded Resonator

When a vibrating system possesses two degrees of freedom which are coupled together through a single coupling mechanism, a biquadratic coupled mode equation for the resonant frequency of the form

$$(f_a^2 - f^2)(f_b^2 - f^2) = \Gamma^2 f_a^2 f_b^2 \qquad (1)$$

can be obtained.

Onoe and Tiersten[1] suggested that the resonances of a rectangular resonator, as shown in Fig. 1, could be represented by this simple equation and would yield a dispersion relation of the type shown in Fig. 2. In this case, as illustrated in Fig. 2, f_a corresponds to the width resonance as $G \to 0$, f_b to the height resonance as $G \to \infty$. There are two further resonaces of interest, $f_d = f_b(1 - \Gamma^2)$ corresponding to the height resonance as $G \to 0$ and $f_c = f_a(1 - \Gamma^2)$ corresponding to the width resonance as $G \to \infty$.

It has been shown by Kino and DeSilets[2,3] that for an open circuited resonator of hexagonal material such as PZT

$$f_a = \frac{1}{2L}\sqrt{\frac{c_{11}^D}{\rho}} \; ; \quad f_b = \frac{1}{2H}\sqrt{\frac{c_{33}^D}{\rho}}$$
$$f_c = \frac{1}{2L}\sqrt{\frac{c_{11}'^E}{\rho}}\left(\frac{2}{\pi}X_c\right) \; ; \quad f_d = \frac{1}{2H}\sqrt{\frac{c_{33}'^D}{\rho}} \qquad (2)$$

where

$$c_{11}^D = c_{11}^E\left(1 + \frac{e_{z1}^2}{\varepsilon_{zz}^S c_{11}^E}\right) \; ; \quad c_{33}'^D = c_{33}'^E(1 + K'^2), \qquad (3)$$

$$c_{33}^D = c_{33}^E\left[1 + \frac{e_{z3}^2}{\varepsilon_{zz}^S c_{33}^E}\right]$$

0090-5607/80/0000-0989 $00.75 © 1980 IEEE

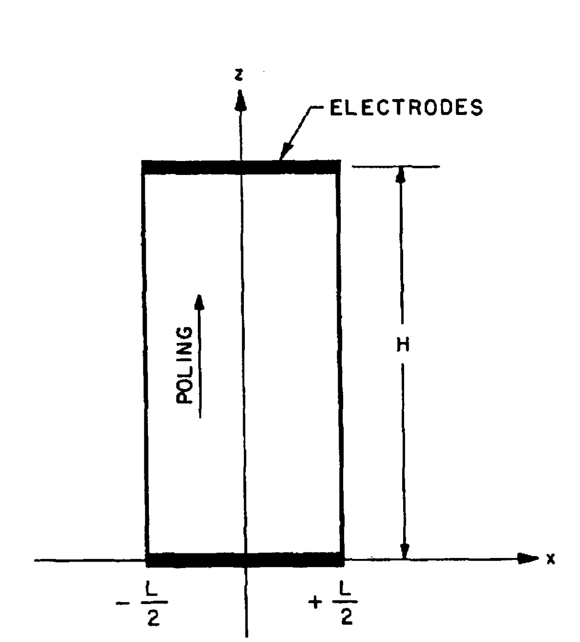

Figure 1 - Cross section of a typical array element

$$K'^{2} = \frac{e'^{2}_{z3}}{c'^{E}_{33}\varepsilon'^{S}_{zz}} \; ; \; c'^{E}_{33} = c^{E}_{33}\left[1 - \frac{(c^{E}_{13})^{2}}{c^{E}_{11}c^{E}_{33}}\right] , \tag{4}$$

$$c'^{E}_{11} = c^{E}_{11}\left[1 - \frac{(c^{E}_{13})^{2}}{c^{E}_{11}c^{E}_{33}}\right] \; ;$$

$$e'_{z3} = e_{z3} - \frac{e_{z1}c^{E}_{13}}{c^{E}_{11}} , \tag{5}$$

and

$$\varepsilon'^{S}_{zz} = \varepsilon^{S}_{zz} + \frac{e^{2}_{z3}}{c^{E}_{11}} \; ; \; e'_{z1} = e_{z1} - \frac{e_{z3}c^{E}_{13}}{c^{E}_{33}} . \tag{6}$$

X_c is the first, non-zero positive root of

$$\frac{\tan X_c}{X_c} = \frac{-1}{K^{2}_{c}} \tag{7}$$

where

$$K^{2}_{c} = \frac{e'^{2}_{z1}}{c'^{E}_{11}\varepsilon'^{S}_{zz}} \tag{8}$$

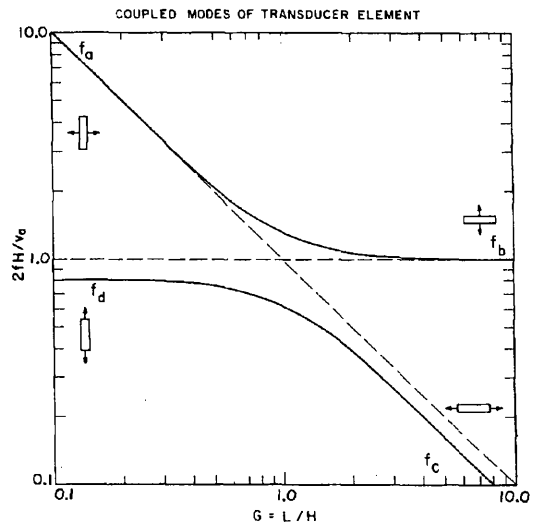

Figure 2 - Theoretical dispersion curves for resonator with two degrees of freedom and a single coupling mechanism

Note the f_c for the open circuit case given here is different from f_c given in the references. A similar set of relations for a short circuited resonator have been shown by Onoe and Tiersten[1] to be

$$f_a = \frac{1}{2L}\sqrt{\frac{c^{E}_{11}}{\rho}} \; ; \; f_b = \frac{1}{2H}\sqrt{\frac{c^{D}_{33}}{\rho}}\left(\frac{2}{\pi} X_b\right)$$

$$f_c = \frac{1}{2L}\sqrt{\frac{c'^{E}_{11}}{\rho}} \; ; \; f_d = \frac{1}{2H}\sqrt{\frac{c'^{D}_{33}}{\rho}}\left(\frac{2}{\pi} X_d\right) \tag{9}$$

$$c^{D}_{33} = c^{E}_{33}\left[1 + \frac{e^{2}_{z3}}{\varepsilon^{S}_{zz}c^{E}_{33}}\right] \tag{10}$$

and X_b and X_d are the first non-zero positive roots of

$$\tan X_b = \frac{X_b}{k^{2}_{t}} \; ; \; \tan X_d = \frac{X_d}{k'^{2}_{t}} \tag{11}$$

where

$$k^{2}_{t} = \frac{e^{2}_{z3}}{\varepsilon^{S}_{zz}c^{D}_{33}} \; ; \; k'^{2}_{t} = \frac{e'^{2}_{z3}}{\varepsilon'^{S}_{zz}c'^{D}_{33}} \tag{12}$$

In order to check this theory, it is necessary to know f_a , f_b , and Γ accurately for both open circuited and short circuited resonators. We have found that the material constants supplied to us by the manufacturer are not always sufficiently accurate. Thus, it has been necessary to carry out a set of three frequency measurements to determine the parameters required.

We first measured the open and short circuited fundamental resonances of a piezoelectric plate with a configuration ratio of the order of 10 to 20. We then determined the open and short circuited resonances of a long strip resonator with a configuration ratio on the order of 4 or 5 and a length about 20 times the width. The third and most difficult measurement was made on a resonator with a very low configuration ratio, with a height 10 times its width. This resonator was constructed from an electroded slab of PZT-5H which was approximately 4 times as thick as the slab used to make the other resonators. This was necessary for the resonator to have a reasonable width of about 0.3 mm, as we found that resonators cut any thinner tend to be depoled. Owing to the increased size and decrease in resonant frequencies of this resonator, the magnitude of the impedance became too great to allow accurate determination of the resonant frequency of the fundamental extensional resonance. This forced us to use the location of the conductance peaks of the first and third harmonics of extensional resonances to accurately determine the extensional resonance frequencies and the electromechanical coupling to this mode.

After determining these limiting frequencies, all the elastic coefficients and piezoelectric coupling coefficients of interest for the material can be calculated. It is also possible to use Eq. (1) to plot both the resonant and anti-resonant frequencies as a function of G, as is shown in Fig. 3. The dots show the measured short circuit anti-resonant frequencies which are in excellent agreement with theory. The circles show the measured open circuit resonant frequencies; these latter results only give a fair fit to this simple theory. We believe the resonators with G less than 0.5 to be partially depoled due to diamond sawing, as their widths were comparable to the grain size of the ceramic, except for the larger lower frequency resonator previously described. The points at G = 0.1 in Fig. 3 correspond to this lower frequency resonator, normalized appropriately. The normalized resonant frequency can be seen to be higher than that of smaller resonators with comparable configuration ratios.

We have calculated from these experimental results the values of k_{eff}^2, the effective electromechanical coupling coefficient, for resonators with configuration ratios from 0.1 to 1.0. Over this range, we can use the equation

$$\frac{\tan X}{X} = \frac{1}{k_{eff}^2} \qquad (13)$$

where $X = (\pi/2)(f_1/f_0)$, f_1 is the short circuit resonant frequency, and f_0 is the open circuit resonant frequency.

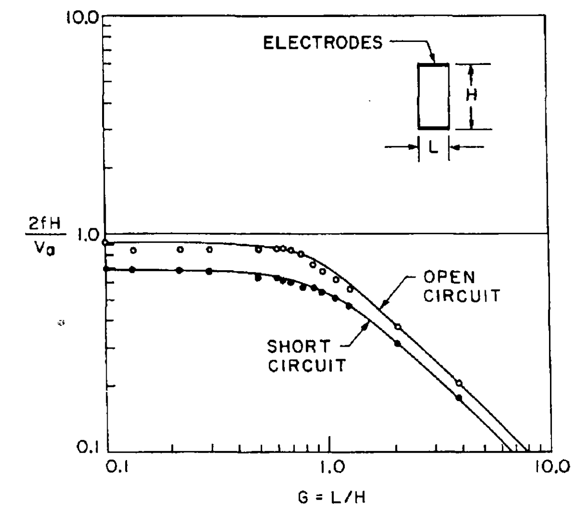

Figure 3 - Experimentally measured resonant frequencies plotted against theory over a wide range of configuration ratios

This calculated electromechanical coupling coefficient for the air loaded resonators is plotted as a function of configuration ratio in Fig. 4. It can be seen that there is a maximum value of 0.52 near G = 0.65. These results tend to agree with other words by Sato et al.[4] and Fabian.[5] Again, the point at G = 0.1 is higher than the points near to it because it did not suffer as much depoling due to sawing.

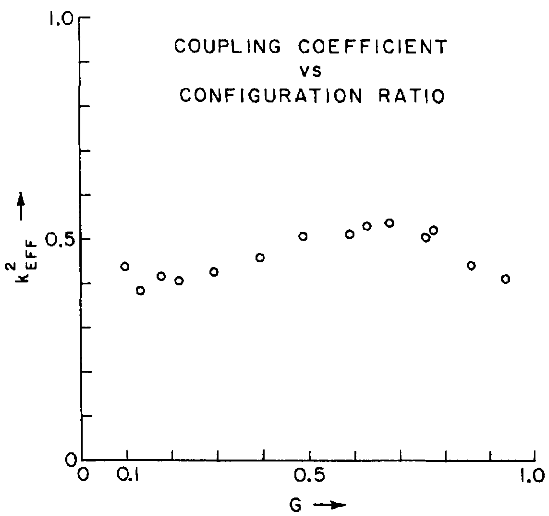

Figure 4 - Measured values of electromechanical coupling coefficients plotted as a function of configuration ratio

It has been an observation that this k_{eff}^2 will decrease as much as 10 to 20% when the resonator is acoustically loaded. Fabian[5] has suggested that this may be due to the lack of symmetry in the resonator, loaded only on one side. More work will be needed to fully understand this effect.

3. The Acoustically Loaded Strip Resonator

Kino and DeSilets[2,3] have developed a variational theory to determine the effective impedance to a narrow strip resonator of a semi-infinite solid or liquid medium. If the strip resonator width is such that $L \ll \lambda$, where λ is the wavelength in the semi-infinite medium, radiating evanescent shear and longitudinal waves as well as Rayleigh waves on the surface of a solid medium can be excited. As the transducer is made narrower it begins to behave like a line source and emit cylindrical waves rather than a longitudinal plane wave. The evanescent waves give rise to reactive loading while the real part of the impedance is determined by the propagating waves. The radiation efficiency into the cylindrical longitudinal wave which is excited by a narrow transducer tends to be poor, so the radiation impedance is lower than than for a wide resonator.

We have carried out the theoretical calculations and experimental measurements for a transducer radiating into water and into a low impedance backing material (epoxy measured has a longitudinal wave of impedance $Z_0 = 3.16 \times 10^6$ kg/m^{2}sec, longitudinal wave velocity $v_L = 2.75$ mm/µsec, and Poisson's ratio $\sigma = 0.37$). The elements in our array are typically $\lambda/2$ wide in water and on the order of $\lambda/3$ wide in epoxy at their center frequency of 3 MHz. The calculated backing impedance is essentially the plane wave impedance, but the impedance in water is complex with a real part quite different from the longitudinal plane wave impedance, even at center frequency.

Figure 5 shows the calculated electrical impedance as a function of frequency for a PZT-5H resonator with a center frequency of 2.86 MHz and width 0.276λ in the water load. The configuration ratio was $G = 0.218$. The broken lines correspond to the calculated impedance using a real acoustic load, corresponding to the acoustical impedance of water. The solid lines correspond to the electrical impedance with the calculated complex acoustic load impedance.

In the experiment, the resonator was loaded acoustically by water on one of the electroded ends. A very thin mylar sheet (12.7µ thick) was laid over a water surface, and the resonator was pressed against the mylar with a coupling material. If water contacted the sides of the resonator, we found that there is a major change in the measured impedance. The experimental results are shown in Fig. 5 as dots. It can be seen that there is excellent agreement between the complex load theory and the experiment. The use of a real load impedance would give very poor results. Similar agreement was also obtained with resonators where $G = 0.594$ and 0.490.

COMPARISON OF THEORY AND EXPERIMENT

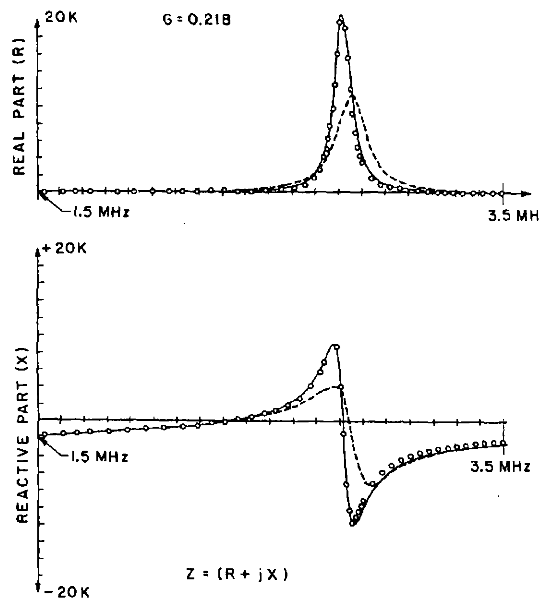

Figure 5 - Experimentally measured values of electrical impedance of a ceramic strip resonator loaded on one side by water are plotted as circles to compare them to theoretically predicted electrical impedances. The broken line shows the electrical impedance we expect to measure if the resonator was loaded acoustically by a real load impedance; the solid line shows the electrical impedance we expect to measure if the resonator is loaded by the complex acoustic load impedance predicted by the variational theory.

A similar set of results is shown in Fig. 6 for an epoxy backed resonator with $G = 0.371$, and 0.274 wavelengths in the epoxy wide at center frequency. Once more there is good agreement between theory and experiment, although the use of the complex load impedance theory does not change the results very much for this width of resonator. Measurements have been made of the electrical impedance with several different configuration ratios. In most cases, there is good agreement between theory and experiment. However, as the configuratio ratio becomes greater than approximately 0.7, the model begins to break down.

Finally, we have used a vitreous carbon matching layer between the resonator and a water load. We model the matching layer as half an unloaded isotropic resonator, assuming the matching layer-PZT interface to be fairly stationary. The frequencies calculated using Eq. (1) are used to obtain the effective impedance and effective transmission line length for the layer. The experimental results for the electrical impedance of the resonator with a matching layer are compared with theory in Fig. 7. Here we see that there is fair agreement between theory and experiment, though this agreement was far more tentative with respect to widths, materials, and configuration ratio than in the previously described experiments.

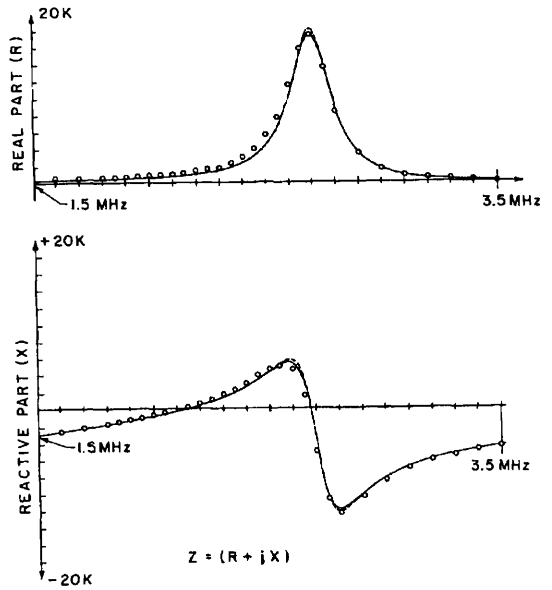

Figure 6 - Same as Figure 5 except the resonator is loaded by epoxy

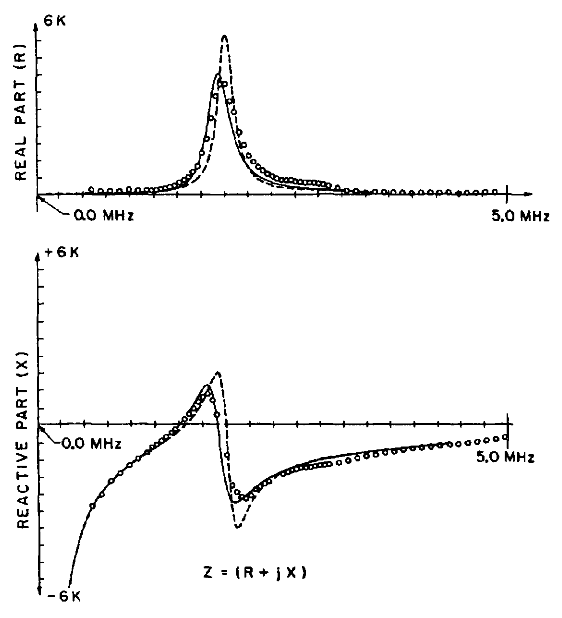

Figure 7 - Same as Figure 5 except the resonator has a matching layer between it and the water load

4. Conclusions

We have used the Onoe and Tiersten theory to calculate the open and short circuit resonances of a short rectangular resonator. The agreement between theory and experiment for a short circuited resonator is excellent but not as good for open circuit resonances. We have used the experimental results to calculate an effective piezoelectric coupling coefficient.

We have taken account of the complex nature of the radiation impedance loading a narrow resonator loaded by water or epoxy. The experiments give good agreement with the complex load theory. The experiments also confirm the theory for a quarter wavelength matching layer.

ACKNOWLEDGEMENTS

This work was supported by the Electric Power Research Institute under Contract No. RP609-1 and by the Air Force Office of Scientific Research under Contract No. F49620-79-C-0217.

REFERENCES

1. M. Onoe and M.F. Tiersten, "Resonant Frequencies of Finite Piezoelectric Ceramic Vibrators with High Electromechanical Coupling," IEEE Trans. on Ultrasonics Engineering, vol. 10, pp. 32-39, 1963.

2. G.S. Kino and C.S. DeSilets, "Design of Slotted Transducer Arrays with Matched Backings," Ultrasonic Imaging, vol. 1, pp. 189-209, 1979.

3. C.S. DeSilets, "Transducer Arrays Suitable for Acoustic Imaging," Ph.D. Thesis, Stanford University, 1978.

4. J. Sato, M. Kawabuchi, and F. Fukumoto, "Dependence of the Electromechanical Coupling Coefficient on the Width-to-Thickness Ratio of Plank-Shaped Piezoelectric Transducers Used for Electronically Scanned Ultrasound Diagnostic Systems," J. Acoust. Soc. Am., vol. 66, pp. 1609-1611, 1979.

5. E.L. Fabian, "Resonant Vibrations of Ceramic Rectangular Bar Transducers," Bell Laboratories Technical Report, Murray Hill, New Jersey, 1961.